Exercise 20

Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies.

(a) Find dF/dr and explain its meaning. What does the minus sign indicate?

(b) Suppose it is known that the earth attracts an object with a force that decreases at the rate of 2 N/km when $r = 20{,}000$ km. How fast does this force change when $r = 10{,}000$ km?

Solution

Part (a)

The derivative of the force (its magnitude) is

$$\frac{dF}{dr} = -\frac{2GmM}{r^3},$$

and it represents the rate that the force increases as the radius increases. The minus sign indicates that as the distance between the two bodies increases (decreases), the gravitational force between them decreases (increases).

Part (b)

If the earth attracts an object with a force that decreases at the rate of 2 N/km when $r = 20{,}000$ km, then

$$\left.\frac{dF}{dr}\right|_{r=20\,000} = -\frac{2GmM}{(20\,000)^3} = -2.$$

Solve for the group of constants GmM.

$$GmM = (20\,000)^3$$

Therefore, when $r = 10{,}000$ km,

$$\left.\frac{dF}{dr}\right|_{r=10\,000} = -\frac{2GmM}{(10\,000)^3} = -\frac{2(20\,000)^3}{(10\,000)^3} = -16\ \frac{\text{N}}{\text{km}}.$$